International Refereed Journal of Scientific Research in Engineering (IRJSRE)
www.irjsre.com Volume 1 Issue 1 ǁ Dec. 2016 ǁ PP. 07-11

Estimation of Metal Recovery Using Exponential Distribution

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ABSTRACT : *The metal recovery is widely used for the assessment of plant performance. It has been formulized as a uniform function in the form of a ratio of output to input. In this study, metal recovery formula in an exponential form was developed based on exponential distribution. The metal recovery in the exponential form was shown with the establishment of probability density functions, based on weights of valuable metal (or mineral) in concentrate and tailing (or slag) depending on weight of valuable metal in feed. The lowest weight of valuable metal in feed was determined using the data over a 1-month period at Elazığ ferrochrome plant in Turkey. Amongst data, only the lowest weight and capacity were re-analyzed over a 6-month period. Results of this study indicate that metal recovery based on the exponential distribution can be used effectively to estimate the lowest weight of valuable metal in feed.*

Keywords: *Metal recovery, ferrochrome, mineral processing, metallurgical process*

I. INTRODUCTION

Performance assessment in a mineral or metallurgical process plant and the control of operation using the evaluated results are quite important. In this study two metal recovery formulae are used to evaluate the plant performance. The first equation has been reformulated using assays according to an exponential distribution [1]. The plant performance can be assessed by calculating the weight of a valuable metal from the weights of feed (F), concentrate (C) and tailing (T) and their corresponding assays f, c and t, respectively. Plant recovery is formulated as [2]:

$$R = \frac{C \times c}{F \times f} \times 100\% \tag{1}$$

since input is equal to output;

$$F \times f = C \times c + T \times t \tag{2}$$

Plant recovery can be rewritten as;

$$R = \frac{C \times c}{C \times c + T \times t} \times 100 \quad \% \tag{3}$$

Metal recoveries obtained from Eqs. (1) and (3) are discrete, i.e. they only give results related to definite time frame of operation and hence it is not possible to make any estimation for the plant. Therefore, if they are reformulated with the use of valuable metal weights, plant performance could be effectively estimated.

II. FORMULIZATION OF THE METAL RECOVERY USING EXPONENTIAL DISTRIBUTION

Relationships between components of two-dimensional continuous random variable are particularly important in probability and statistical analysis. The shaded portion of the random variable (Fig. 1) shows the interference area [3], which is indicative of the weight probability of a valuable metal in feed ($w = F \times f$). As shown in Eq. (2), shaded portion represents a two-dimensional continuous random variable with weights of a valuable metal in tailing and concentrate components. In this case, the probability of mean weight of valuable metal in tailing, $\overline{T} \times \bar{t}$, being less than the weight of valuable metal in feed is given by;

$$P(\overline{T} \times \bar{t} \le w) = \int_0^w f_t(w)dw \tag{4}$$

where, $f_t(w)$ is the probability density function of weight of valuable metal in tailing.

Probability of mean weight of valuable metal in concentrate, $\overline{C} \times \bar{c}$, being greater than the weight of valuable metal in feed is given by

$$P(\overline{C} \times \bar{c} \geq w) = \int_{w}^{\infty} f_{c}(w) dw \quad (5)$$

where, $f_c(w)$ is the probability density function of weight of valuable metal in concentrate.

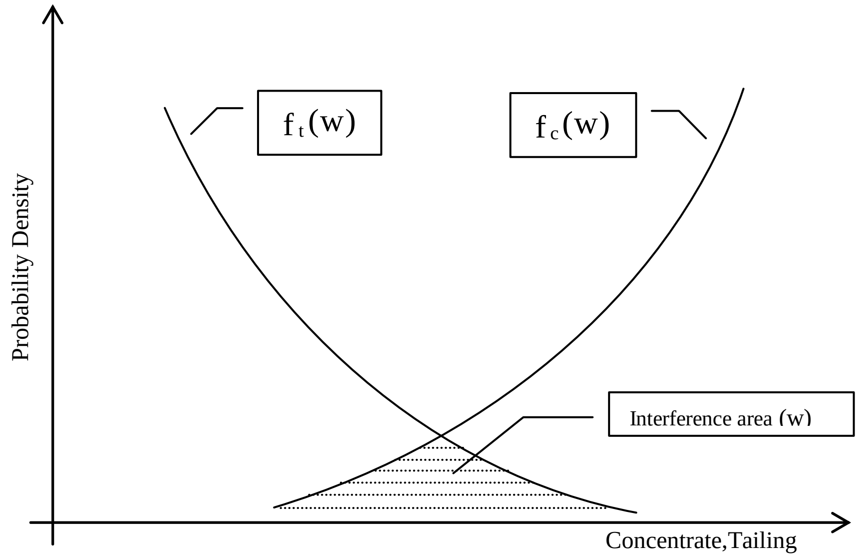

Fig. 1. The interference diagram for the interference area.

Since weight of valuable metal in concentrate and weight of valuable metal in tailing are assumed to be independent random variables, the probabilities of two independent variables are multiplied [4]. Hence, the metal recovery, R, for the possible values of weight of valuable metal in feed is defined as:

$$R(w) = P(\overline{T} \times \bar{t} \leq w) \times P(\overline{C} \times \bar{c} \geq w) \quad (6)$$

Probability density functions are replaced into eq. (7).

$$R(w) = \int_{0}^{w} f_{t}(w) \times \left[\int_{w}^{\infty} f_{c}(w) dw \right] dw \quad (7)$$

Input and output variables (such as feed, concentrate and tailing) in plant recovery formulation are expressed with only one parameter. In this case, it complies with the exponential distribution which has only one-parameter probability density function. First $\overline{C} \times \bar{c}$ and $\overline{T} \times \bar{t}$, mean weights of valuable metal in concentrate and tailing, then valuable metal rates in concentrate λ and tailing μ per unit weight are calculated:

$$\lambda = \frac{1}{\overline{C} \times \bar{c}} \quad (8)$$

$$\mu = \frac{1}{\overline{T} \times \bar{t}} \quad (9)$$

If probability density functions derived from exponential distribution are written as weight of valuable metal in feed:

$$f_{c}(w) = \lambda \times e^{-\lambda \times w} \quad (10)$$

$$f_{t}(w) = \mu \times e^{-\mu \times w} \quad (11)$$

Since the weights of valuable metal in concentrate and tailing should be $\infty \geq \overline{C} \times \bar{c} \geq w$ and $0 \leq \overline{T} \times \bar{t} \leq w$, probability density functions of concentrate and tailing are integrated as in Eq. (7).

$$R(w) = \int_{0}^{w} \mu \times e^{-\mu \times w} \left[\int_{w}^{\infty} \lambda \times e^{-\lambda \times w} dw \right] dw \quad (12)$$

$$R(w) = \frac{\mu}{\mu + \lambda} - \frac{\mu}{\mu + \lambda} \times e^{-(\mu + \lambda) \times w} \quad (13)$$

When the weights of valuable metal are replaced into Eq. (14):

$$R(w)=\frac{\overline{C}\times\overline{c}}{\overline{T}\times\overline{t}+\overline{C}\times\overline{c}}-\frac{\overline{C}\times\overline{c}}{\overline{T}\times\overline{t}+\overline{C}\times\overline{c}}\times e^{-\left(\frac{\overline{C}\times\overline{c}+\overline{T}\times\overline{t}}{\overline{C}\times\overline{c}\times\overline{T}\times\overline{t}}\right)\times w} \quad (14)$$

III. CASE STUDY: ESTIMATION OF METAL RECOVERY OF A FERROCHROME PLANT

In this study, Elazığ ferrochrome plant-A in Turkey is investigated. This plant was established in 1977 with a high carbon ferrochrome production capacity of 50,000 tons per year. There are two arc furnaces with a power of 17 MWA each [5]. The metallurgical balance for this plant over a 1-month period was tabulated (Table 1). The mean weights of valuable metal in ferrochrome and slag, obtained from the ferrochrome plant over a 1-month period, were placed into Eqs. (8), (9) and (13).

$$\lambda=\frac{1}{\overline{C}\times\overline{c}}=\frac{1}{103.22\times 0.6357}=0.01524$$

$$\mu=\frac{1}{\overline{T}\times\overline{t}}=\frac{1}{185.79\times 0.0289}=0\cdot 18624$$

$$\mathrm{R(w)}=\frac{0.18624}{0.18624+0.01406}-\frac{0.18624}{0\cdot 18624+0.01406}\times \mathrm{e}^{-(0\cdot 18624+0.01406)\times \mathrm{w}} \quad (15)$$

Table 1. The mean values of observation over a 1-month period.

Items	Mean weights (tons)	Mean Assays Cr (%)	Mean Metal Cr Weights (tons)
Ferrochrome	103.22	63.57	65.62
Slag (or Tailing)	185.79	2.89	5.37

Eq. 16 is plotted for the possible values of weights of valuable metal in the feed (Fig. 2). It can be seen that the plant recovery in one-month period does not change for feeds containing 27 tons and higher metallic chrome. However, the plant recovery decreases rapidly for feeds containing less than 13 tons of metallic chrome. In order to run this plant at a calculated recovery, at least 39.1 tons or higher daily ferrochrome production capacity per day should be achieved.

Table 2. Values of minimum production capacity over a 6-month period.

Days	Ferrochrome Production		Slag	
	Weights (t)	Assays (%)	Weights (t)	Assays (%)
1	70	64	126	3.64
2	75	64.66	135	3.89
3	85	64.88	153	3.14
4	75	63.44	135	3.7
5	85	64.05	153	3.21
6	85	62.97	153	2.58
7	55	61.4	99	2.2
8	45	61.5	81	3.4
9	55	62.2	99	2.8
10	65	65.1	117	3.39
11	80	63.25	144	2.2
12	85	64.62	153	4.59
13	85	65.6	153	7.38

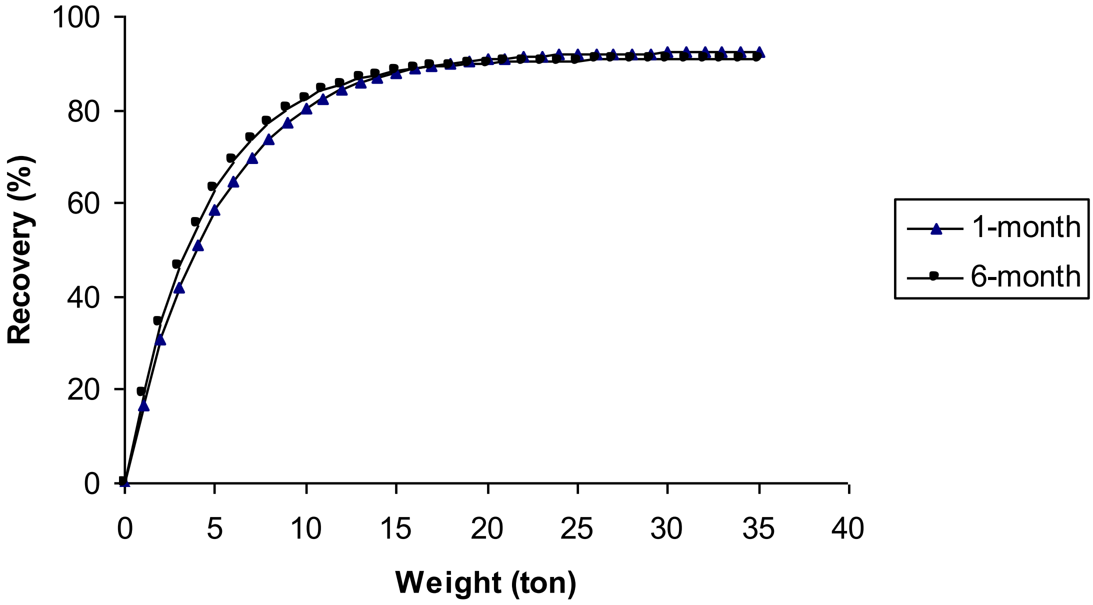

Fig. 2. Curves of recovery versus weight of valuable metal in feed.

In order to investigate the situation mentioned above, the lowest ferrochrome production amounts were collected over a 6-month period (Table 2). The lowest daily ferrochrome productions were observed at 45 and 85 tons. The metallurgical balance for the lowest productions was tabulated over a 6-month period (Table 3). The mean weights of valuable metal in ferrochrome and slag, obtained from the ferrochrome plant over a 6-month period, were placed into Eqs. (8), (9) and (13).

$$\lambda = \frac{1}{\overline{C} \times \overline{c}} = \frac{1}{72.69 \times 0.6367} = 0.02161$$

$$\mu = \frac{1}{\overline{T} \times \overline{t}} = \frac{1}{130.84 \times 0.0355} = 0 \cdot 21529$$

$$R(w) = \frac{0.21529}{0.21529 + 0.02161} - \frac{0.21529}{0.21529 + 0.02161} \times e^{-(0.21529 + 0.02161) \times w} \qquad (16)$$

Table 3. The mean values of observation over a 6-month period.

Item	Mean weights (tons)	Mean Assays Cr (%)	Mean Metal Cr Weights (tons)
Ferrochrome	72.69	63.67	46.28
Slag (or Tailing)	130.84	3.55	4.64

The graph of possible values of valuable metal weights in the feed is shown in Fig.2. Although daily ferrochrome production of plant decreases by 30.53 tons (Table 3), the recovery does not show a considerable change (Fig. 2). Both calculations performed using the suggested recovery formula do not also indicate a significant difference. As a result, plant performance could be better assessed applying the suggested recovery formula.

IV. CONCLUSION

In this study the plant recovery based on definitions and equations of the exponential distribution was developed for improving its use in an easily applicable manner to the mineral and metallurgical processing plants. It was found that the improved plant recovery could be used in determination of minimum production capacity of the plant at a required recovery. It is also possible to estimate the accepted recovery at any weight of valuable metal in the feed.

A case study for the implementation of recovery formula was tested using the historical data from the Elazığ Ferrochrome Plant collected over a 1-month period. The minimum daily ferrochrome production capacity of the plant was found to be higher than 39 tons ferrochrome per day at a required recovery. It was determined that the recovery obtained at the lowest capacity over the 6-month period is very close to the recovery estimated with the suggested recovery formula if the plant is operated under control. In conclusion, it is suggested that improved recovery formula could be used as an efficient tool for the assessment of plant performance.

REFERENCES

[1]. Ankara, H., 2010, A Suggested Improvement to The Plant Recovery Formula: A Case Study for Elazig Ferrochrome Plant. Asian Journal of Chemistry, 22(3), 2073-2080.

[2]. Wills, B. A. and Napier-Munn, T. J., 2006, Wills' Mineral Processing Technology, United Kingdom: Elsevier Science & Technology Books.

[3]. Kapur, K. C. and Lamberson, L. R., 1977, Reliability in Engineering Design, USA, Jhon Wiley & Sons Inc.

[4]. Kara, I., 2000, Olasılık, 4th ed., Turkey, Bilim Teknik Yayınevi (in Turkish)

[5]. Ayten, E., 2001, Determination of Lost Costs in Production Period by Using System Reliability in Elazığ Ferrochrome Plant, M.Sc. Thesis, Osmangazi University, Eskişehir, Turkey (in Turkish, unpublished).